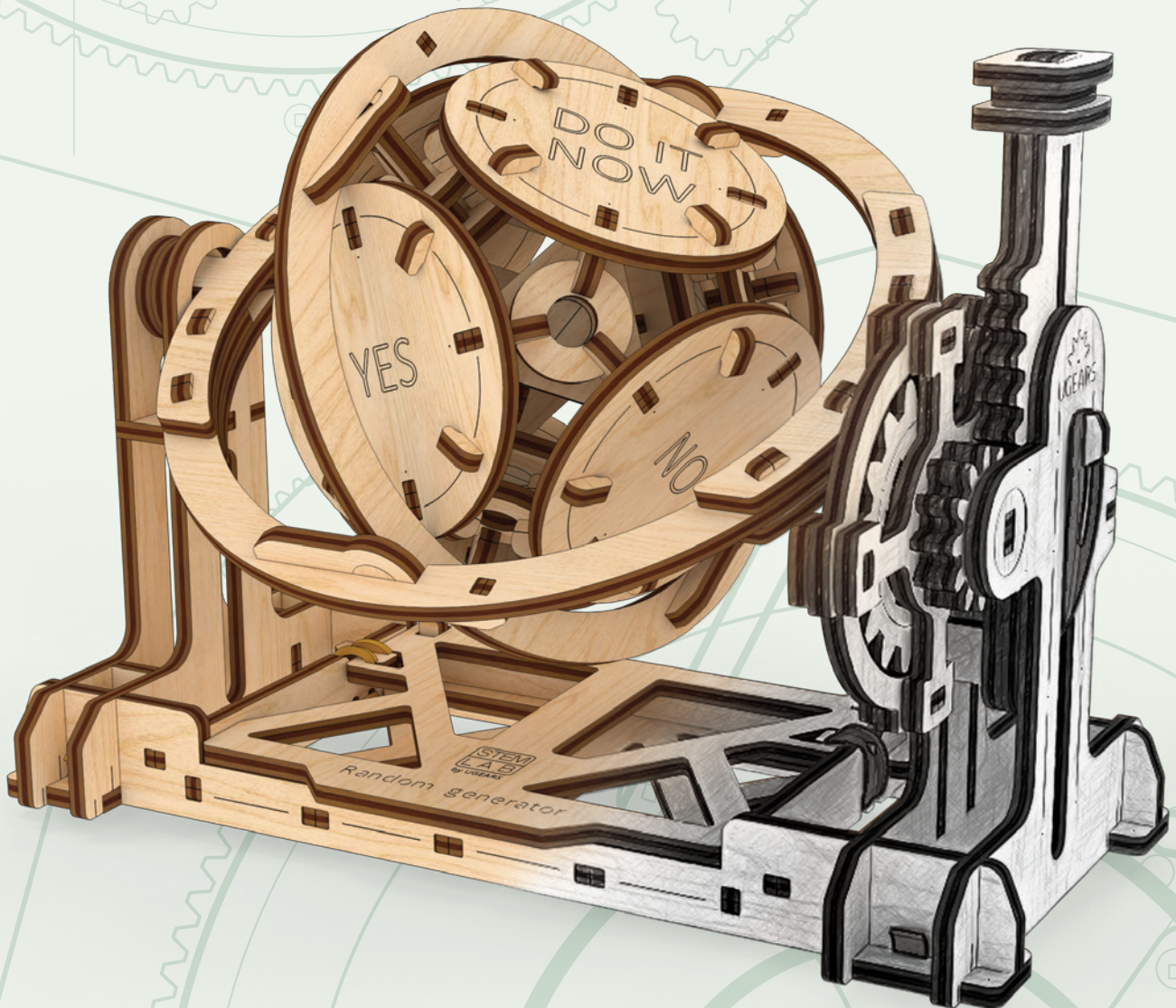


MECHANICAL MODEL

# RANDOM GENERATOR





# §1

## Introduction



### ■ Probability of an Event or Randomness

Coin tossing is one of the oldest, simplest, and most widespread ways of making a random choice.

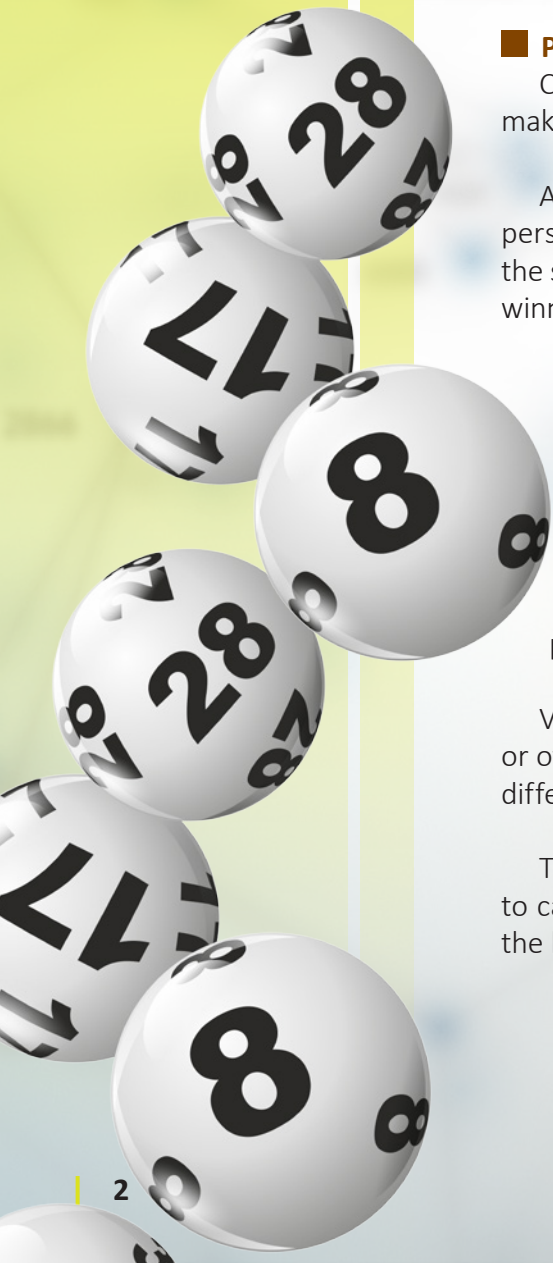
A coin has only two sides: heads (often depicting the head of a prominent person) and tails (the reverse side), so the probability of getting one of them is the same, or equal. Tossing a coin therefore allows for a fair, random selection of winner and loser.

For example, a referee tosses a coin before a football match starts to determine which team will bring the ball into play.

«Drawing lots» is a means of selecting among several people or contestants. Each contestant puts a paper or other such item with their name on it into a bowl or other container. An item or «lot» is then randomly chosen to determine a winner, establish sequential order among the group, or set the pairings of contestants in a sports tournament.

Various items may be used to draw lots for giveaways like raffles, lotteries, or other such events: cards or sheets of paper, sticks of different lengths or with different markings, dice, balls with numbers, etc.

To understand the draw, or your odds of winning, you first need to know how to calculate the probability of an event. For that, let's become acquainted with the basic concept of probability theory.





# §2

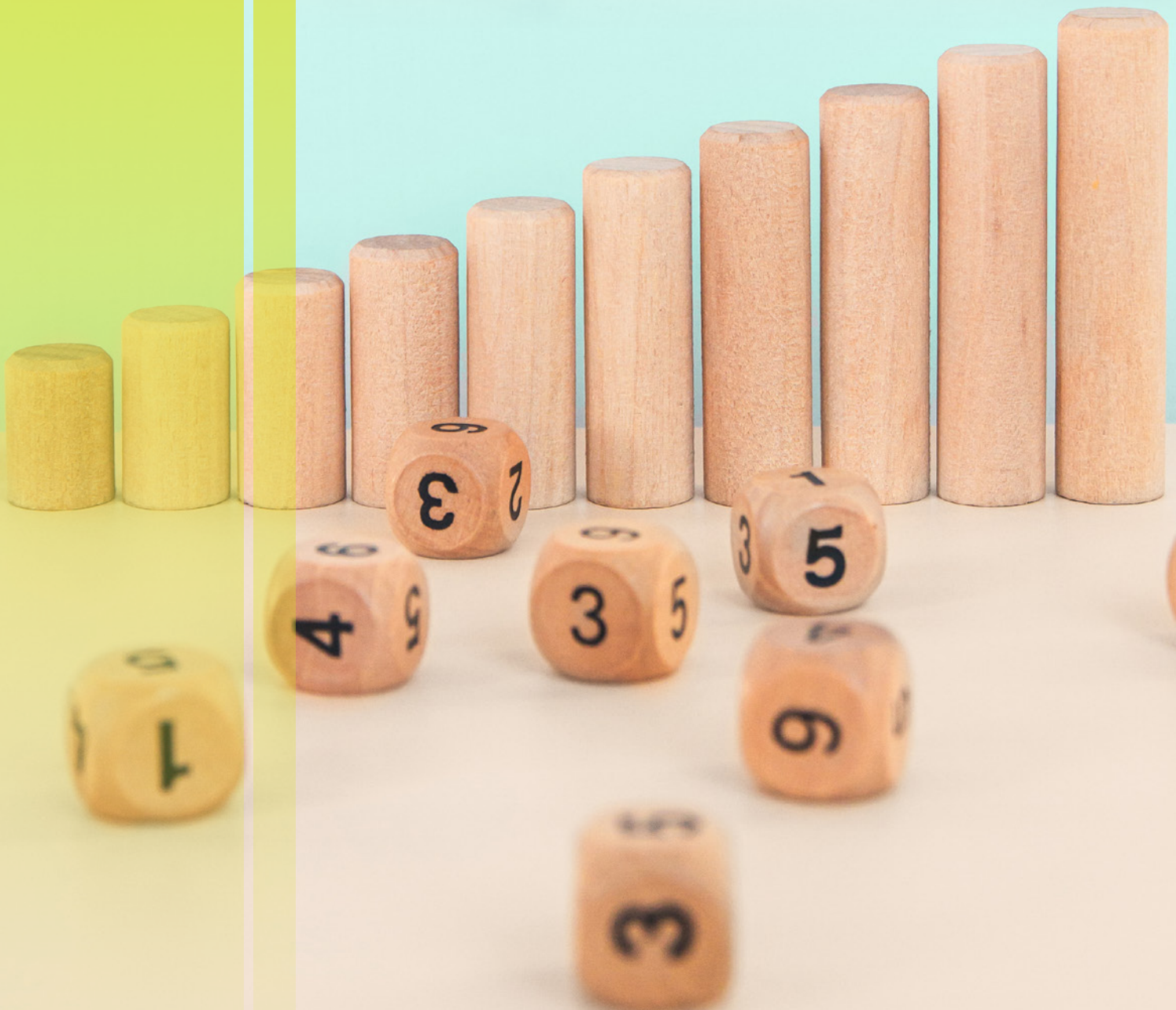
## The History behind Probability Theory

Probability theory is actually a relatively modern branch of mathematics, building on the work of Arab mathematicians between the 8<sup>th</sup> and 13<sup>th</sup> centuries.

The development of probability theory largely parallels efforts to understand gambling. It was later used in demographic research, insurance studies, and other applied sciences.

These days, the principles of probability theory can be applied in one manner or another to nearly every area of human activity.

One of the first works of probability theory was Jacob Bernoulli's book, "The Art of Conjecturing" (1713). The Swiss mathematician proposed a classic definition of the probability of a random event.

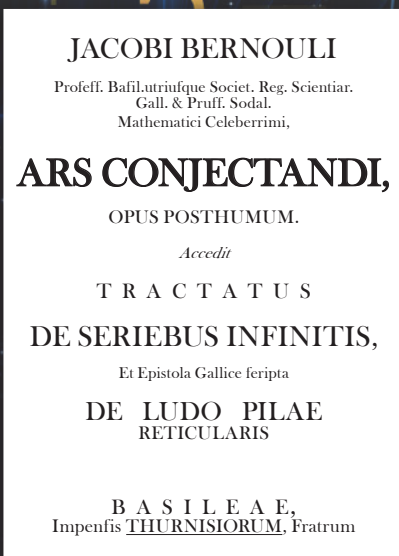


\*Previously, mathematicians frequently did the hard work of crunching numbers to calculate probabilistic outcomes. Historians believe that the helpful substitution of «frequency» for «quantity» (that is, dividing any given outcome by the total number of outcomes) was stimulated by statistical considerations. Specifically, frequency, as opposed to quantity, tends to stabilize as the number of observations increases.





Jacob Bernoulli



Bernoulli's work



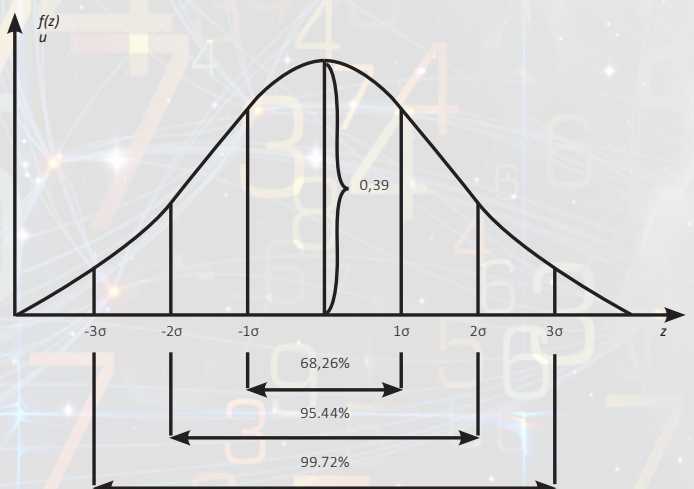
Johann Carl Friedrich Gauss

The definition of probability, as set forth by Bernoulli, immediately gained general acceptance. It was reproduced by Abraham de Moivre in his book, «The Doctrine of Chances» (1718) and by all subsequent mathematicians. The most important clarification was that all «elementary outcomes» must be equally probable. This proposition was made by French mathematician Pierre-Simon de Laplace in 1812. If it is impossible to calculate classical probability (for example, due to inability to identify equally probable outcomes), then Bernoulli suggested using a statistical approach. That is, estimating the probability based on the results from observations of this or related events.

In the first part of his treatise, Bernoulli completely reprinted Huygens' «Treatise on Probability Theory,» to which he gave the highest praise, and substantially supplemented it with his own comments. Bernoulli expounded combinatorics in detail, and used it to solve several problems of random selection. In the last part of the book, which remains unfinished, Bernoulli intended to consider economic and other practical applications of probability theory.

The practical applications of probability theory expanded significantly at the beginning of the 19th century. The concept of probability was also defined for continuous random variables, making it possible to apply methods of mathematical analysis to situations where there is an infinite continuum of possible outcomes.

Carl Friedrich Gauss, the famed German mathematician and physicist, made a significant contribution to the development of probability theory. He was constantly engaged in astronomical calculations and developed a probabilistic method for dealing with measurements containing observational errors (1809). Gauss outlined his final version of the theory in his work, «Theory of the Combination of Observations Least Subject to Error» (1823, 1828). His contribution to the development of probability theory can be seen in «Gauss's Law,» relating to electric fields.



The probability density graph of the normal distribution and percentage of random variable hits on segments equal to the standard deviation.



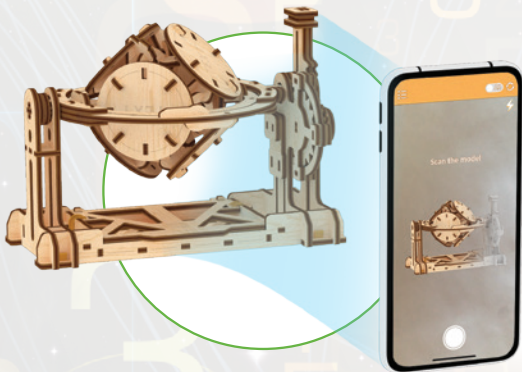
## 1 Scan QR to download App



## 2 Open the application



## 3 Point and align the image on the screen with the model



## 4 Interact in AR



A Ugears STEM-lab mechanical model is an interactive guide to how a mechanism works.

Assemble the RANDOM GENERATOR, learn its key principles and how it works.

Use Ugears AR application that will take you on a trip to augmented reality. Point the camera of your mobile phone or tablet at the assembled model and discover how the mechanism is used in real life. Interact with the model by changing the viewpoint and angle to see how the Random Generator ends up with different results (gets random answers to your questions).



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Should you have any questions about assembly, we are always here for you to suggest the best solution and provide the help you might need. Our 24/7 customer support service will accept and process your request promptly and professionally.

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# §3

## Finding the probability of an event and random number generation



To understand probability theory and how to use it in practice, consider the concept of the probability of an event.

If the event is impossible, its probability is 0.

If the event is inevitable (a certain event), its probability is 1.

If the event is not certain and is not impossible, its probability is between 0 and 1.

When we say that an event is unlikely (for example, winning a large amount in the lottery), it means its probability of occurrence is close to zero. In such a case, you would likely need to make many attempts for the event to occur.

On the contrary, when the probability of an event is high (for example, the probability that an egg will have one yolk rather than two), its probability is close to one. In most cases, the event will occur—when you crack the egg you will find only one yolk inside; only in very rare instances will it have two.

The probability of all events can be characterized by a number ranging from 0 to 1.

For example, the probability of getting heads on a coin toss is 0.5. Thus, the probability of getting tails is also 0.5 since there are no other possible outcomes.

Let's take a closer look at a die (plural: dice) and find the probability of getting one particular face of the six faces on the cube. By definition, the probability of this event is  $1/6$  or 0.167.





So how do you find the number that denotes the probability of an event?

Consider the examples above—the coin and the die. They are both perfectly symmetrical bodies (so equally probable elementary outcomes).

We know for certain that a tossed coin will fall (thus, the probability of this event is 1), and that it will fall on one of two sides (it does not matter which one). Since two events—heads or tails—are equally probable, we divide 1 by 2 and we get a probability of 0.5 for the event.

Similarly, we know that a rolled die will land on one of its faces. The probability of getting any of the six faces (corresponding to numbers from 1 to 6) is equal to 1. Since all events or outcomes are equally probable (the cube is symmetrical), we divide 1 by 6 and get a probability of  $1/6$  or 0.167.

This value means that the probability of getting any particular number, e.g., the face marked with the number 3, is 0.167. But what is the practical use of this number (calculated probability of an event)?

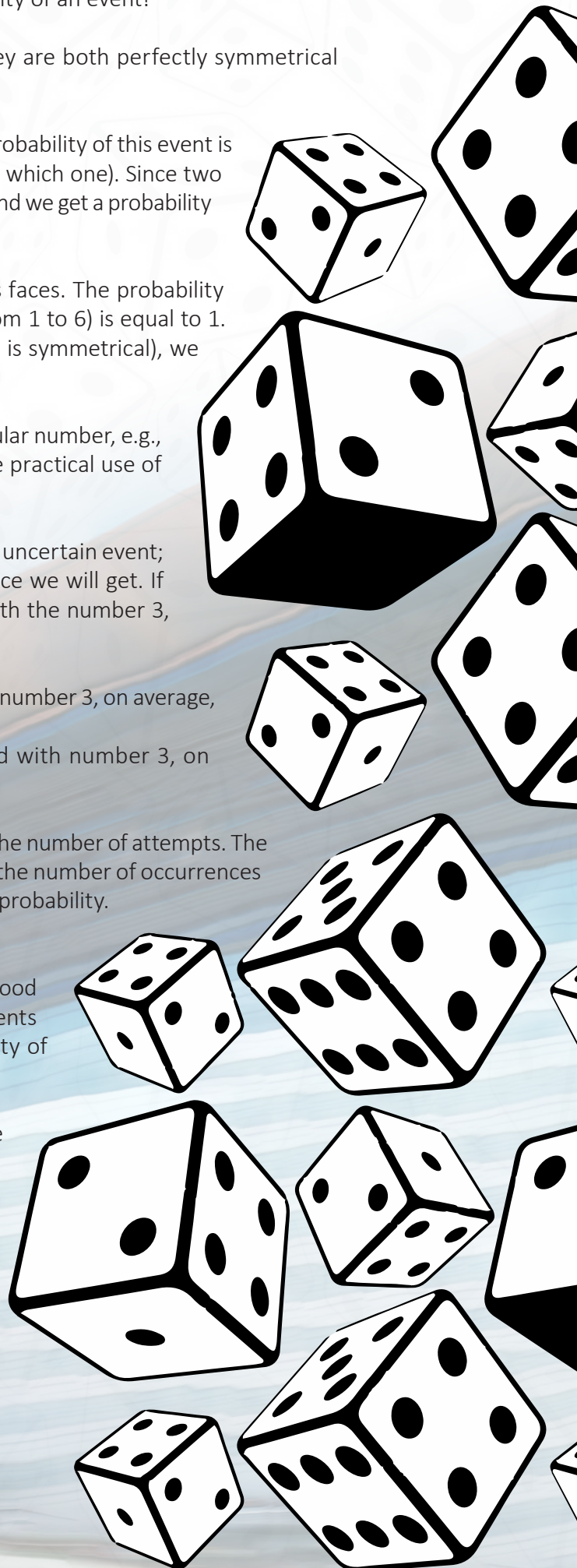
It is impossible to predict with certainty the outcome of an uncertain event; when casting a die, we can never know in advance which face we will get. If we choose a random event, e.g., getting the face marked with the number 3, we can say the following:

- When casting 100 times we will get the face marked with number 3, on average, 17 times
- When casting 1000 times we will get the face marked with number 3, on average, 167 times, and so on...

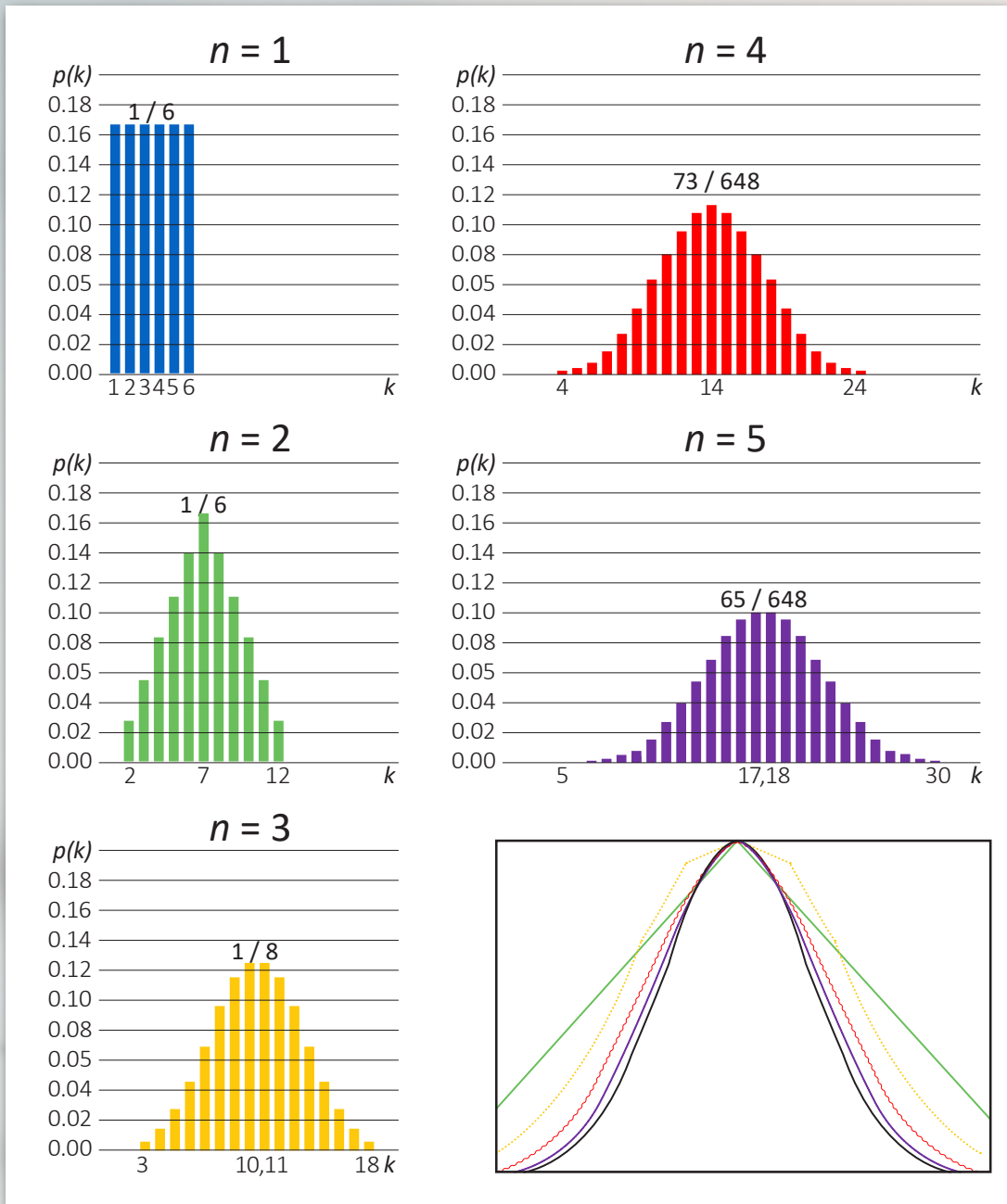
All we have to do is multiply the probability of the event by the number of attempts. The more attempts we make, the closer the observed frequency (the number of occurrences of the face marked with number 3) will be to the theoretical probability.

Probability theory also makes it possible to predict the likelihood of various outcomes where there are several consecutive events or several events that occur simultaneously, if the probability of each is known in advance.

Thus, coins, dice, and other mechanical devices make it possible to conduct a fair drawing of lots—to choose one person or team from many options.







Other mechanical devices can be used for random number generation. Special chips are included in modern computers for this purpose. These chips generate pseudo-random numbers within any range (from 1 to 100, or 1 to a million) and perform these operations very quickly (literally in milliseconds). Pseudo-random numbers are not truly random, as they are determined by a computer algorithm, but their distribution is statistically indistinguishable from numbers that are truly random.



# §4

## The UGEARS Random Generator and its practical uses

The UGEARS Random Generator is a mechanical model that can be used to generate, at random, answers to simple questions. The device features an octahedron with six vertices, each of which has a disc with one of the following inscriptions: YES, NO, LATER, TRY AGAIN, DO IT NOW, BETTER NOT TELL YOU. It can be used as an amusing fortune-telling or advice-giving device, similar to the popular «Magic 8-Ball» children's toy.

Because the discs on the vertices are of the same size and weight and are located symmetrically with respect to center mass, any given inscription is equally likely to occur.

Remember that in order to estimate the number of occurrences of an event during a certain number of attempts, you have to multiply the probability of the event by the number of attempts made.

For example, during a series of 60 attempts with the UGEARS Random Generator, the YES sign will appear 10 times on average. If you were to run 600 attempts, the YES sign would appear 100 times on average, and so on. Moreover, the more attempts we carry out, the less discrepancy there will be between the observed frequency and the theoretical probability.

Let's return to the concept of the probability of an event and find the probability that we will get any one of the six vertices of the octahedron. We know that under the influence of gravity, one of the vertices will certainly appear, so the probability of this event is 1.

Since the octahedron is symmetrical and has the ability to rotate freely around its center of gravity, the probability that we get any one of the vertices is the same. Thus, to find the probability of getting any particular vertex among the six vertices, we need to divide 1 by 6. As a result, we get a probability of  $1/6$  or 0.167.





# §5

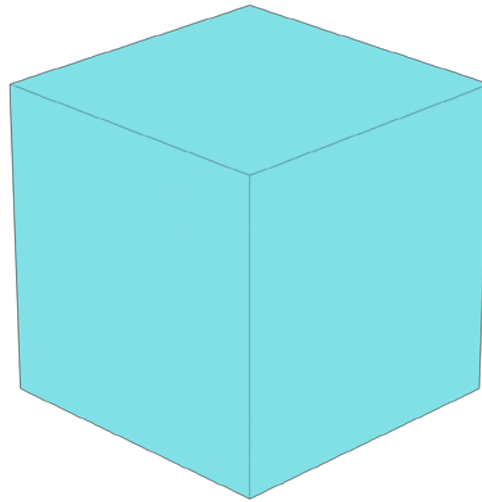
## Mechanical UGEARS Random Generator

In order to determine how the device works, we must turn to geometry.

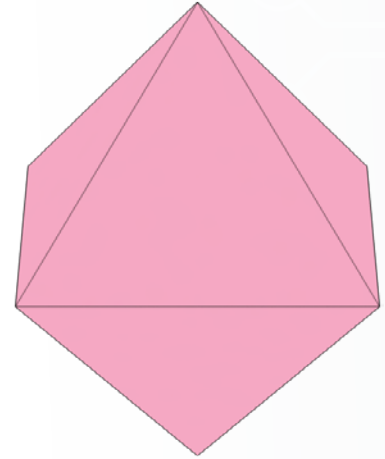
Let's look at two regular polyhedra: a cube and an octahedron.

A cube is a regular polyhedron, all of whose faces are squares. It consists of 6 faces forming 8 vertices.

An octahedron is a regular polyhedron, all of whose faces are equilateral triangles. It consists of 8 faces forming 6 vertices. (see Fig. 4)



Cube



Octahedron

Fig. 4

A cube and an octahedron are dual to each other. If you mark the midpoints of the square faces of the cube, then these points will be the vertices of the inscribed octahedron. Conversely, if you mark the midpoints of the triangular faces of the octahedron, then these points will be the vertices of the inscribed cube (see Fig. 5)

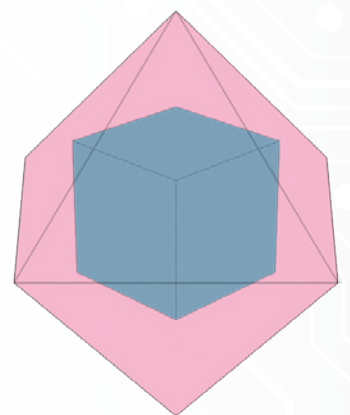
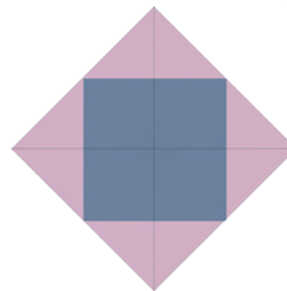
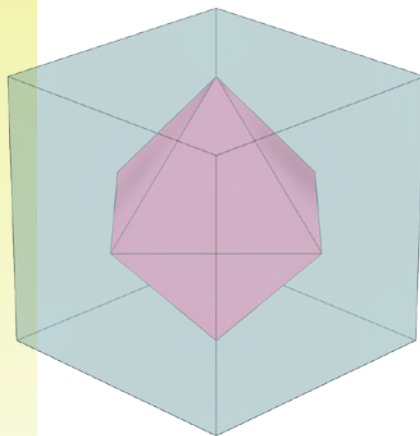
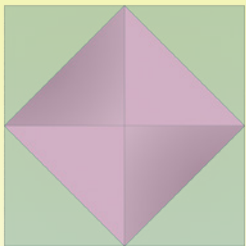


Fig. 5

Imagine an octahedron that is empty. Let's construct this octahedron of a lightweight material (for example, wood), and place inside it a small, heavy iron ball. The weight of the ball is much greater than the weight of the octahedron, while the size of the ball is much smaller than the size of the inner cavity of the octahedron.



Now let's take a look at the design of the UGEARS Random Generator. Inside, there is an octahedron with a weighting agent (iron ball). The ball is smaller than the octahedron, which allows it to roll freely inside when the octahedron turns. When the octahedron is set in motion, the ball bounces around inside until it eventually settles into one of the octahedron's six hollow vertices (formed by the walls of adjacent faces).

At this point the ball has achieved a stable position in space and within the octahedron, causing the octahedron to come to rest under the influence of gravity, with the ball-laden vertex pointed downwards.

A gimbal (a pivoted support that permits rotation of an object about an axis) gives the octahedron the ability to rotate freely after a strong push (produced with the help of a special lever).

The strong impetus sets the octahedron spinning, allowing the metal ball to move freely inside. As friction overcomes inertia, slowing the spin of the device, the ball settles into one of the vertices (random selection), and the vertex assumes a downward orientation, as described above. The opposing vertex, with its inscribed disc, now appears at the top of the device, giving the randomly-generated answer to the question!

Let's take a deeper look at the mechanical principles underlying the gimbal. As you may already know, a free body in space has 6 degrees of freedom: 3 degrees of free movement along 3 axes (x, y, z); and 3 degrees of free rotation around these three axes. These axes can be placed arbitrarily in space but must be perpendicular to one another.

In order for the octahedron to rotate freely in space, you simply need to provide it with the ability to rotate around three mutually perpendicular axes, and these axes of rotation must pass through its center of mass (coinciding with the geometric center). Otherwise, the weight of the octahedron located above the axis of rotation and the weight of the octahedron located below will be different, and gravity will prevent it from rotating freely around one or more axes.

To make the octahedron capable of rotating around one axis, you need to install it in a frame between two swivel hinges (see Fig. 6) such that the axis of rotation of both hinges is the same and passes through the center of gravity of the octahedron.

Congratulations—we have made the octahedron capable of rotating freely around the first axis!

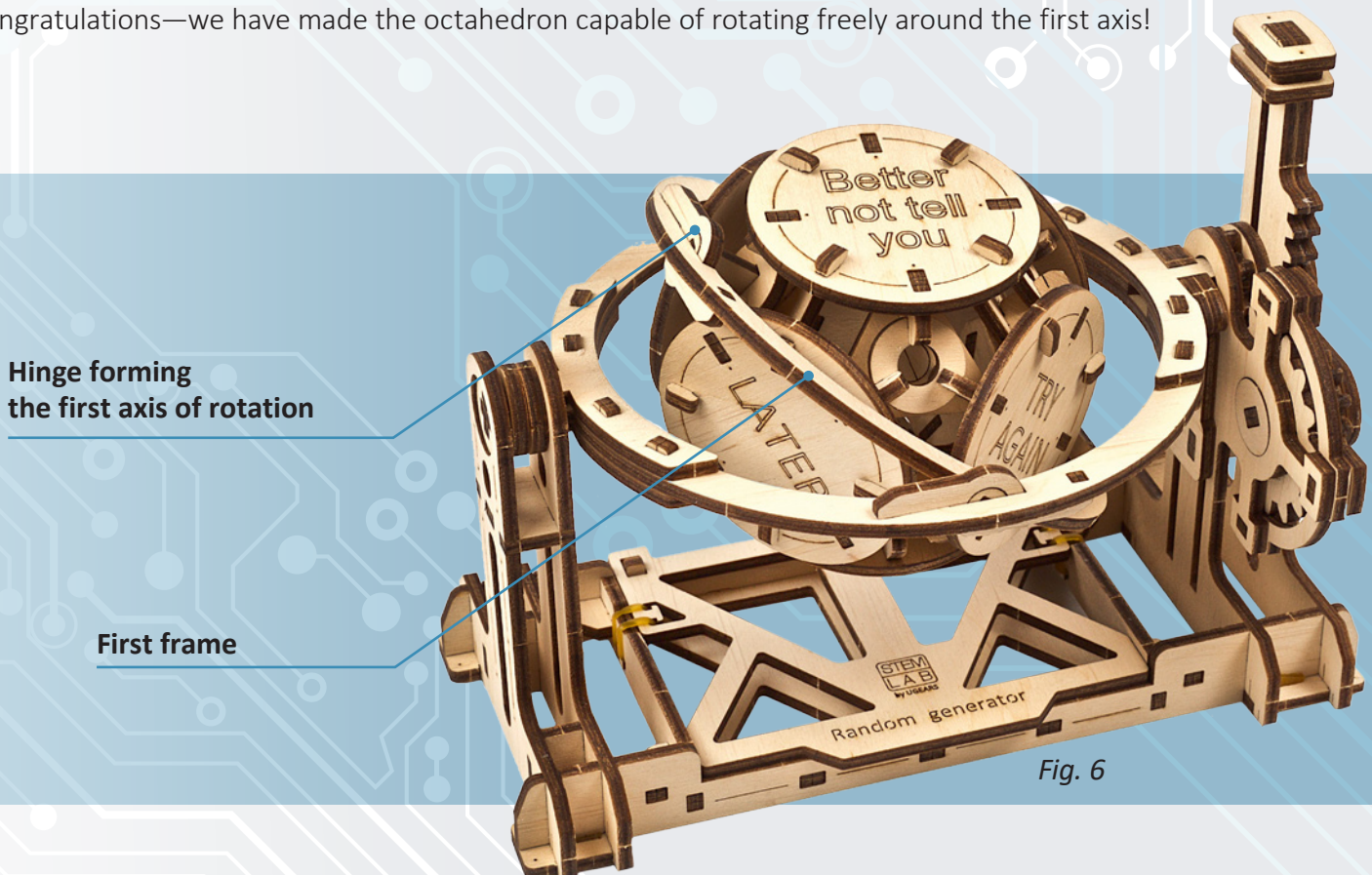


Fig. 6



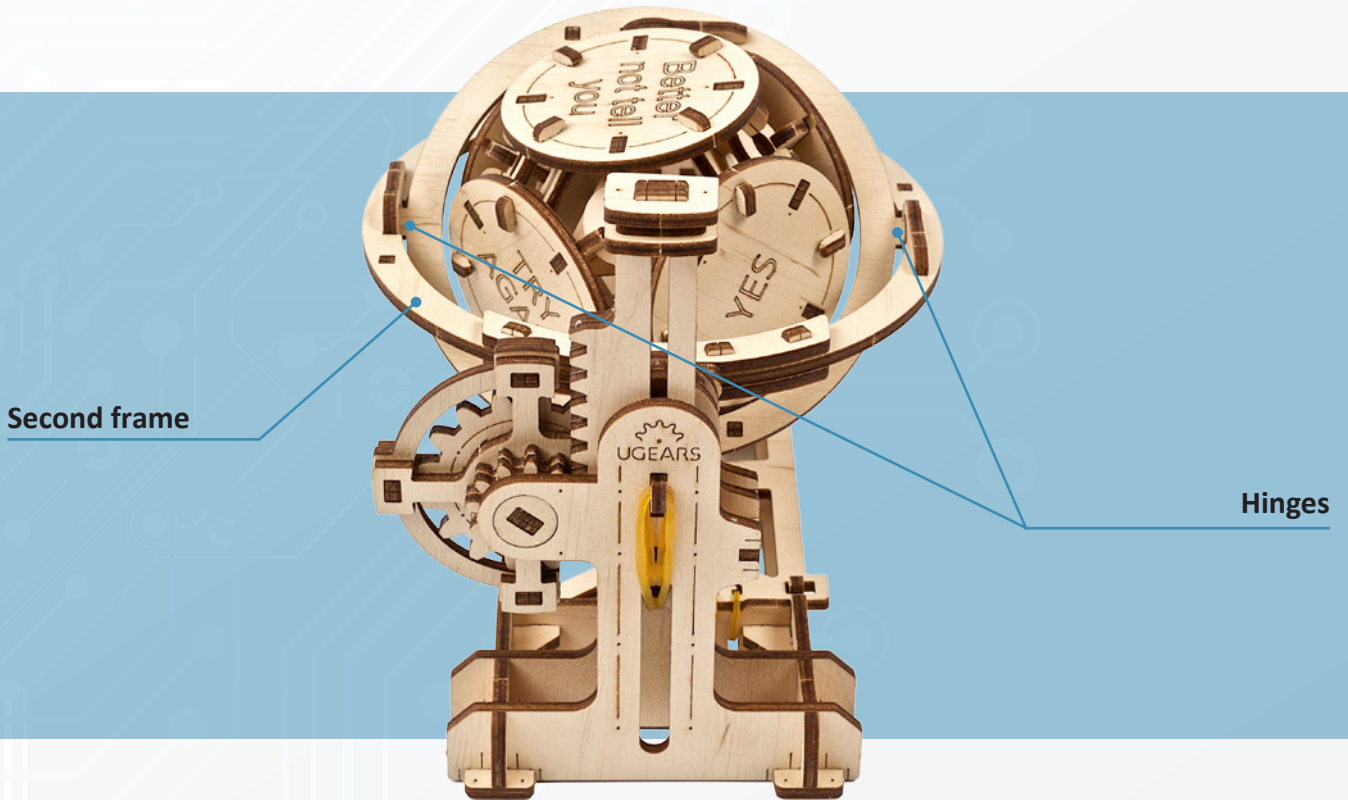


Fig. 7

But how can you make the octahedron capable of simultaneously rotating around another axis?

Well, to do that you need to place the first frame into the second frame and connect them with swivel hinges so that the axis of rotation of this second pair of hinges is the same, and also passes through the octahedron's center of gravity (see Fig. 7). Note that this second axis of rotation should be located at a 90° angle from the first axis.

In order to make the octahedron capable of rotating around the third axis (perpendicular to the first two axes), you need to connect the second frame to a third frame in the same way, again using swivel hinges whose axis of rotation passes through the center of gravity of the octahedron, and at a 90° angle relative to the other two axes (see Fig. 8).

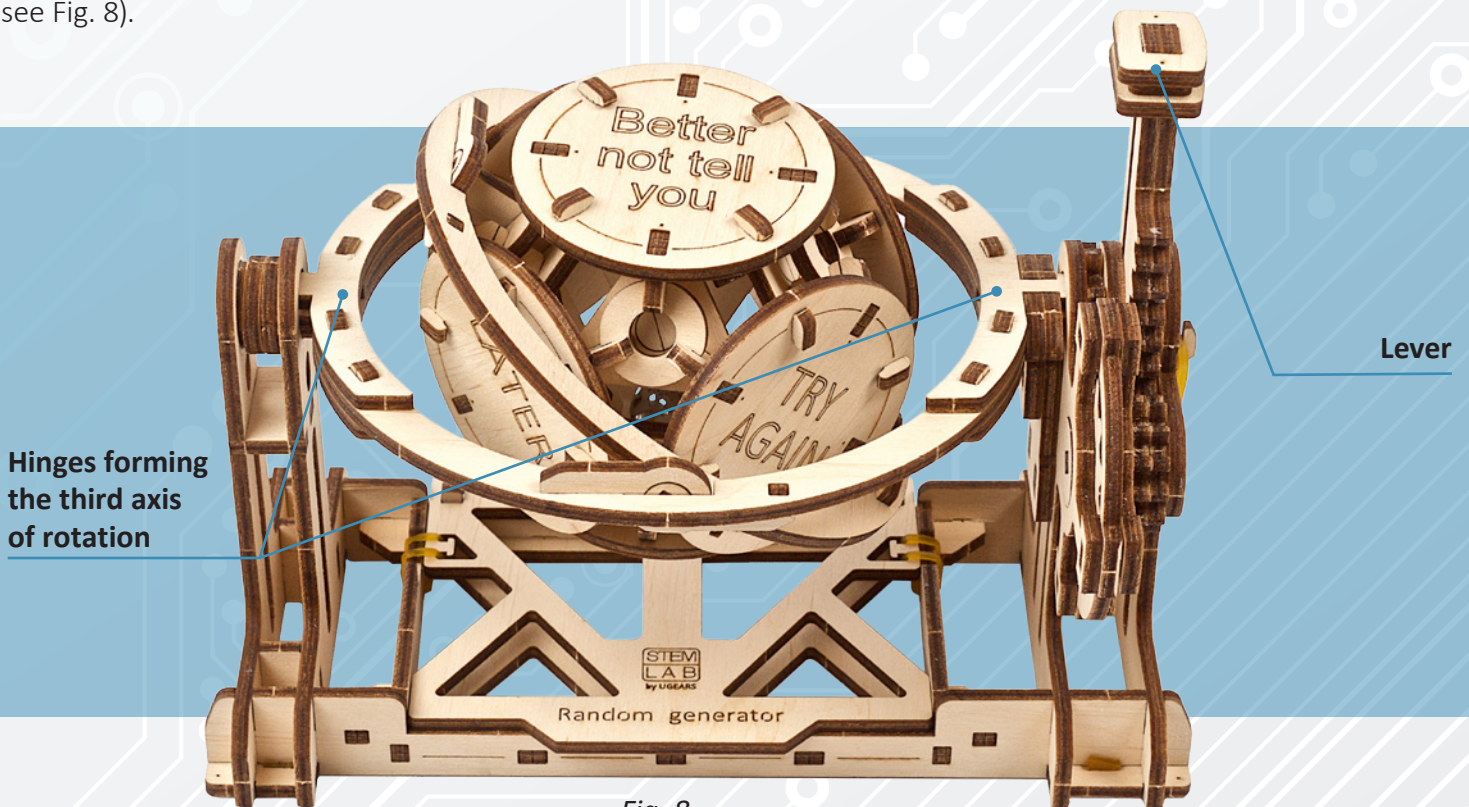


Fig. 8



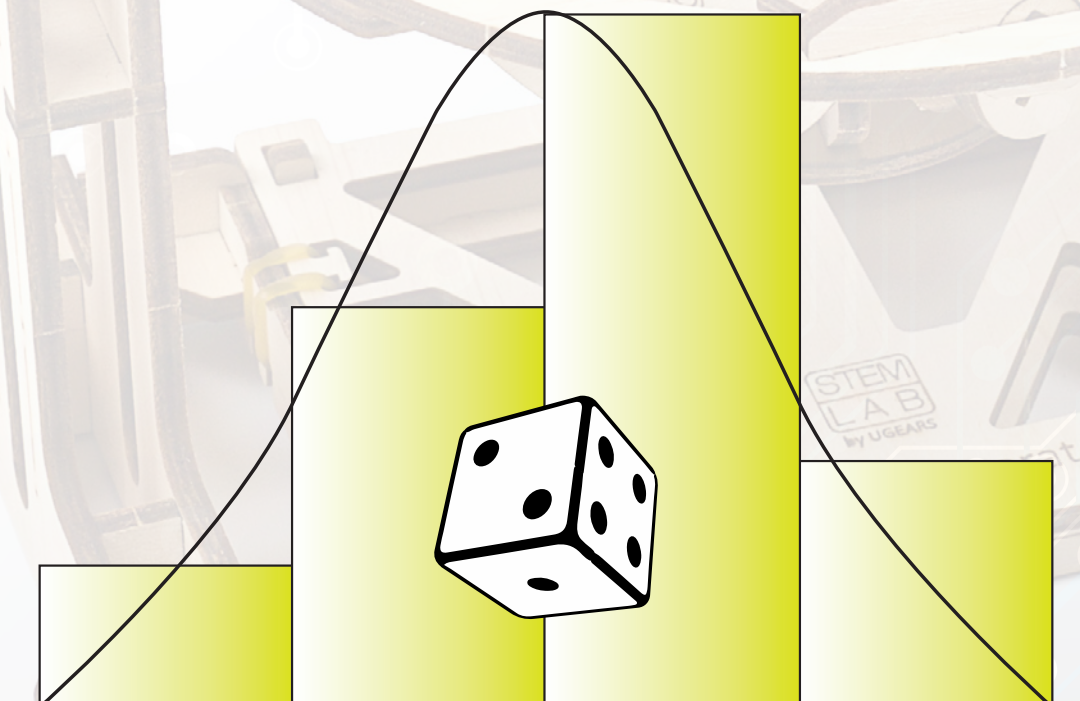
Thus, the UGEARS Random Generator is designed so that the hollow octahedron is capable of freely rotating in space around three mutually-perpendicular axes.

To set the octahedron in motion, you simply press a special lever (see Fig. 8) which uses a gear transmission to create a sharp rotation of the second frame around the third axis of rotation. Under this strong initial impetus, the octahedron rotates simultaneously around all three axes.

Once friction overcomes inertia the octahedron can come to rest in any spatial orientation. The weight of the metal ball, which comes to rest in any of six vertices, ensures that one of the vertices of the octahedron will always face bottom.

Because all six axes of rotation pass through the center of gravity of the octahedron, the probability of the ball coming to rest in any particular vertex is equal, and there is an equal chance that any of the six «answers,» or inscribed discs, will appear on top.

Congratulations—you now understand how to calculate the “probability of events” and how the UGEARS Random Generator works!





# §6

## Test your knowledge

### TEST

#### 1. What does the numeric probability of an event indicate?

- a) The number of attempts needed to make a random event occur
- b) The relation between the expected number of occurrences of an event and the number of attempts made
- c) The number of occurrences of the event

#### 2. What is the range of the probability of an event?

- a) From 0 to 10
- b) From -1 to 1
- c) From 0 to 1

#### 3. What is the probability of an impossible event?

- a) 0
- b) 1
- c) 10

#### 4. To roughly calculate how many times a random event is likely to occur, knowing the number of attempts and the probability of this event,

- a) Find the sum of the number of attempts and the probability of the event
- b) Divide the number of attempts by the probability of the event
- c) Multiply the number of attempts by the probability of the event

#### Task 1.

Imagine that the octahedron in the UGEARS Random Generator is replaced by an icosahedron (a regular polyhedron with 20 equilateral triangle faces, having 12 vertices) (see Fig. 9).

How many different answers could such a device give?

What is the probability of getting any particular one of the answers?

#### Task 2.

Imagine a die in the shape of an icosahedron. When you toss this die, one of the faces will appear at the top. Each face of the icosahedron has its own unique number (numbers are not repeated) starting from one (1,2,3...). What is the probability of getting the face containing the number 1?

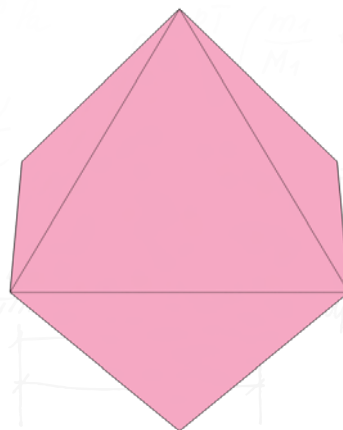


Fig. 9

**Congratulations! You made it!**

Thank you for joining us on this adventure. We hope you had fun and learned a thing or two!