# STEM LAB <br> by UGEARS 

## MECHANICAL MODEL <br> CURVIMETER



Young Engineer's Handbook

§2

## Maps and scales

Topography is a science that studies the shape and character of the Earth's surface. The data for topographic maps comes from land, aerial, or sometimes even space surveys.

A topographic map is a type of map that provides a detailed representation of a certain area. It is characterized by large scale detail and quantitative representation of "relief," i.e., the difference between highest and lowest elevation. Knowing how to interpret a relief map allows the map reader to know what kind of terrain they are facing, e.g., whether there are rivers, mountains or hills, where the roads go, or even how deep the creeks are in order to know whether you will have to swim or can wade across.

The place of any object on the map is found through coordinates: numeric values of latitude, longitude, and elevation.

Topographic maps use the Cartesian coordinate system, first introduced by the French mathematician and philosopher René Descartes in 1637. It specifies a point in space by giving a set of coordinates, which are the distances to the point from the intersection of perpendicular lines called coordinate axes-a horizontal axis ("x") and a vertical axis (" $y$ ") which cross at the origin, whose value is 0 (pic 2 ).


Pic 2. Cartesian coordinate system

Roads are rarely completely straight, in the real world or on a map. Map distances along a curved line or along a route with twists and turns can be difficult to measure.

Straight lines are easily measured with a ruler. There are no tricks here: just place your ruler along the line on the map, measure the distance, and multiply by the number provided in the scale to find out the real world distance.



But what if your route has zigzags or curved lines? Two ways to measure are using a ruler and compass, or a string.

Compass method. Open the legs of a compass 1 cm , set one leg at the starting point, then measure the distance by "walking" the compass along the line (pic. 3). Simply add the number of steps to get the map distance in centimeters, then multiply by the map scale to get the real world distance. You can adjust the step of the compass according to the curvature of the line you want to measure, or if there are frequent turns or zigzags-use smaller compass steps for more curved or winding roads.

String method. Take a piece of string or thread and neatly place it along the route, so that the string follows your route exactly on the map. Be sure to securely fix the string at the beginning point so that it doesn't move. Once the string is laid out along the route, you can cut or mark the string at journey's end on the map. Now simply straighten the string and measure it with a ruler; this is your distance on the map. All that's left to do is multiply by the scale.

These are perfectly acceptable, even clever ways to measure a winding route on a map. However, there is a less complicated and more accurate way to do this using a special tool called a Curvimeter (pic. 4). With a Curvimeter you can simply roll your device along the route and the device will conveniently measure the distance in centimeters as it goes, giving you a final number at the end which you then multiply by the scale.

The Curvimeter is the perfect tool to measure curved lines on a map. These devices go by a variety of names: opisometer, meilograph, or simply a map measurer, but the idea and component parts are the same: a convenient handle, an indication face, and a small wheel to roll along your map route.



Pic 3. Measuring lines by steps of a compass


Pic 4. Measuring lines with a curvimeter

Precise measurements are very important for calculating correct distances.
Measurement error is the difference between a measured quantity and its true value. It characterises the accuracy of measurement.

Of the various means and methods for measuring distances on a map, measurement error will be greatest when curves are measured with a straight ruler. Using a compass or the string method will somewhat reduce measurement error, but these methods can be cumbersome. A curvimeter provides the smallest possible error and the most elegant solution, as the tracing wheel precisely follows every turn.

One thing to keep in mind is that distances calculated from a map will always be shorter than actual travel distances if there are significant changes in elevation along the way. If the area you are going to explore has hills, mountains, and hollows, all that up and down along your route adds distance.

## Once you've measured the length of the route on a map, it's time to hit the road!

## Real World Measurements

Scaled maps represent real world distances. But how were those real world distances calculated in the first place?
There are different ways to measure distances in the real world. You can choose one method or another depending on the accuracy desired. Just as with map measurements, real world measurements will have a greater or lesser degree of measurement error depending on the method used.

Steps. The average length of a human step is 0.75 m . Measuring distance in steps can be used when a larger margin of error is acceptable.

Field Compass. In Western Europe, people sometimes used a so-called "field compass," a scaled-up version of the familiar desk tool, to measure relatively short distances. Some surveyors and farmers still use this instrument to this day. A field compass is an A-shaped tool that usually has 1 or 2 meters distance between the legs. On the ground, it works pretty much the same as its little twin on the map: a surveyor sticks one end of the compass in the ground at the starting point then "walks" the legs of the field compass along the path while counting the compass's steps (Pic 5a).


Pic. 5. Measuring distance in the field

Surveyor's wheel. Also called a clickwheel, hodometer, waywiser, trundle wheel, measuring wheel or perambulator, a surveyor's wheel works in very similar fashion to a curvimeter, and unsurprisingly, it is a very precise way to measure distance (Pic 5b).

A surveyor's wheel is a large curvimeter designed for use on the ground. The diameter of the wheel is 318.47 mm , and its circumference is exactly 1 m . This means that one full revolution of the wheel measures one meter of length along your path. The device is usually equipped with a counter registering the number of revolutions, which typically resets at the 1000 m mark. The counter in a surveyor's wheel works exactly the same as the Counter model in Ugears STEM collection (to learn more, check the Counter model out!).


Surveyor's wheel design by Raymond F. Martin, Jr., and Arthur W. Enslein, January 17, 1952


## Curvimeter. Historical reference and how it works

Curvimeter is a device for measuring the lengths of curved lines, used mostly to find distances on maps and charts.

There is no verified information about who invented the curvimeter, or when. Some sources suggest it originated in China, others believe its birthplace was ancient Rome or Greece. Some credit its invention to the Roman engineer Vitruvius, who described a very similar device around 23 $B C$. Others credit the Chinese scholar Zhāng Héng.

The first curvimeter patent belongs to the English engineer Edward Russell Morris, who in 1873 registered "a novel device for measuring distances."

## Curvimeters and their functionality.

A curvimeter can be mechanical or digital, but the main functioning principle remains the same: a rubberized measuring wheel is used to measure distance.

A mechanical curvimeter comprises a wheel, a handle and a counter face.

A curvimeter may have one or two faces. Devices with two faces are used to measure distances in different units, e.g., centimeters on one counter face and inches on the other (see "Units of length" below). To calculate distance, run the wheel along your map route and multiply the length counted by the scale of the map.


An illustration of a curvimeter patented by Heberlein and Boss in 1878

Digital curvimeters are a bit more sophisticated. These also have a wheel and a handle but the measured distance will be shown on a digital screen. Moreover, once you input the scale, the device automatically converts measured distance to distance on the ground, and can provide results in kilometers, miles or other units.

Digital curvimeters are more accurate on average than their mechanical counterparts, with a 0.2 percent margin of error for digital compared to a margin of error for mechanical of around 0.5 percent.

How does one measuring wheel provide two different indications? As with a clock, the hands of a curvimeter can rotate with different speeds. For example, the hand on a metric scale will show 10 cm , while the inch hand will have only gone 4 in .

Engineers use special gear arrangements to achieve varying speed of rotation (Pic. 6).

Meshed gears transfer motion to each other. (Pic. 6).
There are drive gears and driven gears.
Drive gears transfer motion to the driven gears.
Motion is transferred between drive and driven meshed gears according to the gear ratio.

Gear ratio $i$ is the ratio between the number of the driven gear cogs to the number of the drive gear cogs.

As you can see on pic. 6 , if $Z_{1}$ is the number of cogs on a drive gear $A$ and $Z_{2}$ is the number of $\operatorname{cogs}$ on a driven $\operatorname{cog} B$, the gear ratio $i$ will be found as follows:

$$
i=\frac{z_{2}}{z_{1}}
$$



Pic 6. Gear arrangement
(meshing gears example)

A unit of length is a reference standard for measuring length and distance.

From ancient times, people have needed to measure distances: the size of a plot of land, shipping routes, roads between towns or to the market, etc. Early travellers, merchants, sailors, builders, and astronomers required a standard reference on which to base their measurements, and tools to make the measurements. Necessity is the mother of invention, so the first measurement units were quick to appear. Old units of length would seem very strange today, as many were based on human body parts.

In ancient Greece, Rome, and Egypt there were units of length such as a finger and the distance one could travel in a day. Olympic athletes raced a stadion, the circumference of a typical sports stadium of the time, which we now calculate to be approximately 192.27 meters.

Emperor Huangdi was the first to introduce a single measuring system in ancient China, which was very similar to the Japanese one, with similar written characters.

The U.S. and a few other countries use miles to measure distance. The mile originated in ancient Rome, and consisted of a thousand paces as measured by every other step-i.e., the total distance after the left foot has hit the ground 1,000 times. Since then, the mile has gone through many interpretations, its size varying from 0.58 km in Egypt to 11.3 km in an old Norwegian system.

By the XIII century, the European system of distance measurement had become a total mess, with 46 competing length units called miles.

Of course, there were also other confusing, non-standard units of length. For example, a pipe was the distance covered by a boat while a sailor smoked a pipe. In Japan distance was measured by horseshoes: the distance a horse made before its straw shoe wore out. Many countries had the practice of measuring distance by the range of a shot arrow. And in Siberia they had buka, the distance at which you could no longer clearly see the two horns of a bull separately.

Needless to say, all these units of length were very approximate and none-tooconvenient. A single, standard way to measure distance was needed.

## Metric System

Developing commercial relations among countries forced people to attempt to unify measuring systems. In the XIV-XVI centuries, merchants sought unified units in their areas of occupation. This is how such units as the inch appeared, measured as the length of three barleycorns.

Scientists from different countries did their best to translate local units and avoid inaccurate measurements. Toward the end of the XVIII century a new units system was created in France. The Metre Convention, also known as the Treaty of the Metre, was signed in Paris on 20 May 1875 by 17 nations.

On the facade of the Ministry of Justice in Paris, just below a ground-floor window, you will find a marble shelf engraved with a horizontal line and the word 'MÈTRE'.

Today, the metric system created in France is an official system of measurements in all countries of the world, except for the USA, Liberia, and Myanmar (Burma). But even those three countries use the metric system when it comes to international trade issues.

Over its first century of use, the metric system evolved into the International System of Units. In 1960, the 11th General Conference on Weights and Measures synthesised the results of a 12-year study into a set of 16 resolutions, named the International System of Units (abbreviated SI from its French name, Le Système International d'Unités). It assumed seven base units, which are the second (time, s), meter (length, m), kilogram (mass, kg ), ampere (electric current, A), kelvin (thermodynamic temperature, K), mole (amount of substance, mol), and candela (luminous intensity, cd).

The International System of Units (SI) is the official system of units of measurement used in nearly every country in the world, in every sphere of life, including science, manufacturing, education and design. Even those few countries using different measurement systems in their daily lives use SI in technical and mechanical fields. Traditional measurement units in those countries have been adapted in a way to be easily converted to SI units using a fixed multiplier.

This extremely important standardization of measurement has become a critical part of every sphere of human activity from engineering to international trade.

## As we mentioned above, one of the base units of SI is a meter (m)

SI multiples of meter
kilometer (km) - ( $1 \mathrm{~km}=1000 \mathrm{~m}$ )
decimeter (dm) - ( $1 \mathrm{dm}=0.1 \mathrm{~m}$ )
centimeter (cm) - $(1 \mathrm{~cm}=0.01 \mathrm{~m})$
millimeter $(\mathrm{mm})-(1 \mathrm{~mm}=0.001 \mathrm{~m})$
In some countries, for example the US, the inch remains the main measuring unit
1 inch $=2.54 \mathrm{~cm}$.

Let's practice calculating gear ratios using the example in Pic 7.
Drive gear $A$ has 16 cogs, $Z_{1}=16$
Driven gear $B$ has 30 cogs, $Z_{2}=30$.
Using the formula $i=\frac{z_{2}}{z_{1}}$ we get the following result:

$$
i=\frac{30}{16}=1.875
$$

The "i" value 1.875 that we calculated means that for every revolution of the driven wheel $B$, the drive gear $A$ will make 1.875 revolutions.

Consequently, you can see that if the number of cogs in a driven wheel is larger than in a drive one, it will always rotate slower, and vice versa. The gear ratio shows how slow/fast the gear wheels rotate in relation to one another.

In order to get a curvimeter to correctly display in both centimeters and inches, we need to ensure the correct ratio between the drive gear connected to the measuring wheel, and the driven gear rotating the hands.

Consequently, gear ratio is the principle on which the correct working of a mechanical curvimeter is based.

Ugears Curvimeter follows this general principle, with a few distinctions that make its design somewhat different.

The gear ratio value i1 used to switch from inches (Measuring wheel) to centimeters (disc II) is 3.28 ( $\mathrm{i}_{1}=3.28$, reflecting there are 3.28 feet in a meter). In order to go from 100 centimeters (the limit of disc II)


Pic 7. Gear arrangement (meshing gears example) to 10 meters (the limit of disc III) the gear ratio value $i_{2}$ is $10\left(i_{2}=10\right)$.

One of the fun characteristics of the Ugears curvimeter is its compact size. The device is designed to be carried around conveniently and easily handled when needed. This is why we use a planetary gear mechanism combined with a cycloid reducer in our curvimeter, to allow this small device to measure relatively large distances.


The planetary gear mechanism gets its name from its resemblance to a solar system that has a star (sun) and planets orbiting it. This gear system features a socalled sun gear at the center and a crown gear at the periphery, connected via satellites (planet pinions) rather than directly. The pinions mesh with the crown gear and transfer the rotation to a pinion carrier. If a planetary gear mechanism is used as a reducer, one of its main elements will be rigidly fixed while the other two work as drive and driven elements.

Therefore, if a crown gear is fixed rigidly, the carrier and sun gear will work as moving parts. If the sun gear is fixed, the carrier and the crown will move. If the carrier is fixed, the crown and the sun will be the ones set in motion when the mechanism starts.

The gear ratio between the drive and driven gears will depend on the number of cogs on each gear and on which of the constructional elements is fixed rigidly. A rigidly fixed crown gear will yield the highest value for the gear ratio.
Let's calculate the gear ratio " $\mathbf{i}$ " using the example from Pic. 8
In this illustration, the crown gear (1) is fixed rigidly, while the carrier (H) and sun gear (3) will move. Let's find the drive and driven bars.

Let's now find the drive and driven elements.
In this case, the sun gear will be a drive element while the carrier works as the driven element.
Now let's determine the rotation speed of the carrier in relation to the rotation speed of the sun gear.

The gear ratio (i) of the sun gear (3) and the carrier (H) with a fixed crown gear can be found using the following
formula: formula:

$$
i_{3 H}^{(1)}=1+\frac{z_{2}}{z_{3}} \cdot \frac{z_{1}}{z_{2}}=1+\frac{z_{1}}{z_{3}}
$$

## where:

$i_{3 H}^{(1)} 1$ is the gear ratio value, (1) means that element 1 is rigid, $\mathbf{3}$ and $\mathbf{H}$ show that the ratio will be the relation of sun gear $\mathbf{3}$ to carrier $\mathbf{H}, z$ is the number of cogs of a particular gear and subscript numbers are the number of the particular element. From the formula we can see that the number of cogs in pinion $\mathbf{2}$ doesn't influence the value of the gear ratio. This is because pinion $\mathbf{2}$ serves as a drive gear in one type of gear arrangement, and as a driven gear in the other.


Crown gear
Large central gear with, cogs on the inside, also known as "the epicyclic gear"


## Pinion carrier

moving element that carries pinion gears sitting on pivots (the pinion gears can rotate freely on their pivots).
The carrier has the same axis as the crown and sun gears.

gears with cogs on the outside (usually 3-6 cogs) Pinions mesh with the sun and crown gears.

Small central gear with cogs on the outside


Pic 8. Planetary gear with 4 bars.
1- Crown (large central gear);
2 - Pinion on moving pivot 3 - Sun gear (small central gear),

## CYCLOID REDUCER <br> (Cycloidal drive)

## Cycloid reducer (Cycloidal drive)

Design-wise, a cycloid reduce resembles a planetary gear mechanism. As with a planetary mechanism, a cycloid drive has four elements. The drive gear rotates in the opposite direction of the driven gear.

Let's find the gear ratio " $i$ " for the mechanism in Pic 9 .

The gear ratio i between drive and driven elements of the cycloid reducer can be found in the following equation:

$$
i=\frac{P-L}{L}
$$

## where:

$P$ is the number of cogs on the crown gear;
$L$ is the number of cogs on the cycloidal

Now that we know what gearing is used in the Ugears STEM LAB Curvimeter, we can take a closer look at the complete design to find out how the gears work together.

## Input shaft

located in the center of the
mechanism, the input shaft drives a cycloidal disc. The shaft has an eccentric bearing, meaning its center is offset (in a planetary gear the sun gear).


Pic 9. Cycloid reducer.
${ }_{7}^{\mathbf{6} \text { - cy cymshoidal disk }}$
7 - cycloidal disk
8-crown disk

- output rollers
$8-$ crown disk
9 -output rollers
110 - output disk


## The Design of the STEM LAB (UGEARS TM) Mechanical Curvimeter

The Mechanical curvimeter model is a measuring tool shaped like a tape measure. The device has a central rubberized tracing wheel (1) with a measuring scale on its face giving the distance in inches.

On its sides, the model has additional scales with planetary and cycloidal mechanisms. One of them is designed to convert the distance to centimeters (disc II), while the other (disc III) tallies measurements in meters and feet. Each scale moves at its own speed.

One rotation of the measuring wheel is 12 inches ( 1 foot). Over that same distance, disc II turns $1 / 3.28$ of its full 1 m circle (reflecting that there are 3.28 feet in a meter). In order to make the conversion from imperial to metric systems of measurement, the curvimeter therefore uses a planetary mechanism with reduction rate $i_{1}=3.28$.

A complete revolution of disc II is 100 centimeters (1 meter) while over that same distance disc III turns $1 / 10$ of its full 10 m circle, displaying 1 meter or 3.28 feet. By employing a cycloidal reducer with $\mathrm{an}_{2} \mathrm{i}_{2}=10$ reduction rate between the second and third discs, the curvimeter is able to measure longer distances (up to a full 10 m ) before resetting.

## There are three scales available in the Ugears Curvimeter:

- Scale I: inches. Displayed on both sides of measuring wheel 1. A complete rotation of wheel 1 is 12 inches or 1 foot.
- Scale II: centimeters. Displayed on both sides of measuring disc II. A complete rotation of disc II is $\mathbf{1 0 0}$ centimeters (1 meter).
- Scale III: meters and feet. Displayed on both sides of measuring disc III. A complete rotation of disc III is 10 meters or 32.8 feet.


Planetary mechanism consists of:

- central sun gear $2\left(z_{2}=25\right)$
- three pinion gears $\mathbf{3}^{2}\left(z_{2}=16\right)$ on pivots
- rigid crown gear $4\left(z_{4}=57\right)$
- pinion carrier 5, with pinion gears (unlike the standard planetary mechanism, in the Ugears curvimeter the axles of the pinion gears are fixed inside the gears and the carrier has holes to accommodate them. This feature was necessitated by the nature of the material the device is made from and doesn't affect the functioning of the mechanism).
- measuring disk II

During measuring, the rubberized wheel $\mathbf{1}$ drives the central sun gear $\mathbf{2}$ and further translates the motion to the pinion gears 3 .

The pinion gears $\mathbf{3}$ freely rotate along the inside of the crown gear $\mathbf{4}$ and drive the carrier $\mathbf{5}$.
The carrier 5 in turn meshes directly with measuring disc II which indicates distance in centimeters.
As mentioned earlier, the reduction ratio of the planetary mechanism can be found using the formula:

$$
i_{25}^{(4)}=1+\frac{z_{4}}{z_{2}}=1+\frac{57}{25}=3,28
$$

## where:

$i_{25}^{(4)}$ is the reduction rate ratio, subscript (4) indicates that the gear $\mathbf{4}$ is fixed rigidly,
subscripts $\mathbf{2}$ and $\mathbf{5}$ indicate that the reduction rate is a ratio of the gear $\mathbf{2}$ to gear 5,
$z$ is the number of cogs in the gears: central sun gear $\mathbf{2}\left(z_{2}=25\right)$ and rigid crown gear $\mathbf{4}\left(z_{4}=57\right)$.


Planetary gear train

Cycloid reducer consists of:

- eccentric camshaft 6;
- cycloidal disk 7, mounted loosely on the eccentric camshaft;
- output rollers 9 ;
- rigid crown ring 8;
- output disk 10, connected to measuring disk III.

The eccentric camshaft $\mathbf{6}$ is mounted on the axis of the planetary mechanism and rotates with it. The camshaft translates the motion to the cycloidal disk. The cycloidal disk $\mathbf{7}$ moves along the inside of the crown ring $\mathbf{8}$ and through the output rollers $\mathbf{9}$ drives the output disk $\mathbf{1 0}$ (measuring disk III).

The reduction ratio of the cycloid reducer is found in the following formula:

$$
i=\frac{P-L}{L}=\frac{11-10}{10}=\frac{1}{10}
$$

## where:

$P$ is the number of cogs on the rigid crown ring, $P=11$;
$L$ is the number of cogs on the cycloidal disk, $L=10$.

## Measuring with the Ugears mechanical Curvimeter

When measuring distance with the Ugears Curvimeter, hold the device perpendicular to the surface and push down slightly as you roll it along the line you wish to measure, to ensure better contact.

Your measuring results will appear on the scales in inches, centimeters, meters, and feet.
You can measure up to 10 meters or 32.8 feet at a time, which is one full rotation of measuring disc III.
To reset the device, set all the discs to $\mathbf{0}$, starting with wheel 1 , then disc $\mathbf{I I}$, and finally disc III, on both sides of the curvimeter.

The double-sided design of the Curvimeter allows you to make two (or more) separate measurements while the device tallies total measured distance. In order to do this, make a measurement, then reset the measuring scales to 0 on discs II and III on one side only, while leaving the measured distance indications on the other side. Now you can make a new measurement. The Curvimeter displays the new measurement on the side that was reset, while the opposite side keeps the running tally of both measurements.


## OBJECTIVE:

To learn to calculate distances on the ground using the scale on a map.
Task 1. Find the scale of a map from the given length (I) on map and corresponding distance on the ground (L):

The scale can be found as follows: 5 cm : $50 \mathrm{~m}-1 \mathrm{~cm}$ : $10 \mathrm{~m}-1$ : 1000

| $\#$ | I (map) | L (ground) | Scale of the map |
| :---: | :---: | :---: | :---: |
| 1 | 5 cm | 50 m | $1: 1000$ |
| 2 | 2 cm | 200 m |  |
| 3 | 4 cm | 4 km |  |
| 4 | 3 cm | 300 km |  |
| 5 | 30 cm | 150 m |  |

Task 2. Find the distance on the ground $\mathrm{L}=$ ? from the scale of the map and the length of the line on the map (I).

For example: 1: $5000, \mathrm{I}=4 \mathrm{~cm}, \mathrm{~L}=$ ? (where $1 \mathrm{~cm}=50 \mathrm{~m}, 4 \mathrm{~cm}=200 \mathrm{~m}$, therefore $\mathrm{L}=200 \mathrm{~m}$ (i.e., 4 cm on the map corresponds to 200 m on the ground)

| $\#$ | I (map) | L (ground) | Scale of the map |
| :---: | :---: | :---: | :---: |
| 1 | 4 cm | 200 m | $1: 5000$ |
| 2 | 6 cm |  | $1: 25000$ |
| 3 | 3 cm |  | $1: 300000$ |
| 4 | $2,5 \mathrm{~cm}$ |  | $1: 5000000$ |

Task 3. Measure the length of the following rivers on a physical map of the world using the curvimeter (for more accurate results use a smaller scale map).
$\square$ a) Amazon River;
b) Nile;
c) Yangtze.

As you measure the curve on the map make sure you roll the wheel of the curvimeter exactly along the entire line. Multiply your result by the scale of the map to find the actual distance on the ground.

1. What does the scale of a map indicate?a) the degree to which actual distances are reduced;b) the degree to which actual distances are increased;c) how the actual distance changed.
2. What do you call the study of the forms and features of land surfaces?
$\square$ a) topography;b) geography;c) cartography.

## 3. What is a curvimeter used for?

a) to measure curved lines;b) to measure mass;c) to measure surface area.
## 4. An early version of the curvimeter was patented by...

a) Edward Russell Morris;b) Nolman;c) Lomonosov.
## Congratulations! You made it!

Thank you for joining us on this adventure. We hope you had fun and learned a thing or two!

